MEHTODOLOGY FOR SMART INDICES

1. Introduction

Investars uses a “SmartIndex” methodology to quantify the performance of stock ratings provided by research firms. The equal-weighted SmartIndex is constructed for each firm to track research firm’s performance.

2. Assumptions

To construct the indices, the following assumptions are made:

A.1. A three rating system is used: buy, neutral, and sell. Other rating systems are converted to the three-tiered rating system;
A.2. Stock closing prices are used in the calculations;
A.3. Stock shares can be traded with fractions;
A.4. Transaction cost and taxes are not included.

In the SmartIndex method, stocks with buy, neutral, and sell ratings in each firm are classified into three indices:

- Positive position (P) – buy
- Neutral position (Ne) – neutral
- Negative position (N) – sell

At the initial day of the indices, $t_1$, stocks covered by each firm are classified into the above three indices according to their ratings. An initial value, $V_p^{total}(t_1)$, e.g., 100, is assigned to each index. The value distributed to each stock is determined by the following weighting method:

**Equal-Weighted Method**

An equal value is allocated to each stock in the index.

$$V_p^i(t_1) = \frac{1}{N_p(t_1)} \cdot V_p^{total}(t_1)$$

(1)

where:

- subscript: $P \in \{Positive, Neutral, Negative\}$
- $N_p(t_1)$: the number of stocks in the index at $t_1$
- $V_p^{total}(t_1)$: the index value at $t_1$
- $V_p^i(t_1)$: the value allocated to stock $i$ in the index at $t_1$
Given the value of each stock, the number of shares, $N_{P}^{i}$, can be determined by stock prices at $t$. The holdings in the indices will maintain unchanged in the following days if there is no rating change. Then, the index value at any trading day $t$ is:

$$V_{P}^{\text{total}}(t_{k}) = \sum_{i=1}^{N_{P}(t_{k})} V_{P}^{i}(t_{k}) = \sum_{i=1}^{N_{P}(t_{k})} N_{P}^{i}(t_{k}) \cdot P^{i}(t_{k})$$ \hspace{1cm} (2)

Dividends, $D^{i}$, can be included in the index total value.

$$V_{P}^{\text{total}}(t_{k}) = \sum_{i=1}^{N_{P}(t_{k})} N_{P}^{i}(t_{k}) \cdot \left[ P^{i}(t_{k}) + D^{i}(t_{k}) \right]$$ \hspace{1cm} (3)

The dividends are reinvested into the index immediately on the ex-dates. The index is rebalanced so that each stock will receive a portion of the dividends proportional to its current weight in the index.

If the stocks’ ratings are updated, the following rules are used to reclassify stocks in the indices:

**Removing stocks:**
- R1. If a stock’s rating is changed, it is removed from the original index.
- R2. If a stock, which was in an index, is not traded on the market or is not covered by the research firm anymore, it is removed from the corresponding index.

**Adding stocks:**
- R3. If a rating is initiated for a stock, which was not in any index, the stock is added into one index according to the rating.
- R4. If a stock’s rating is changed, it is added into the index corresponding to its new rating.

For example, if a stock is downgraded from “buy” to “neutral” at time $t$, it will be removed from the “P” index and added into the “Ne” index.

Since the stocks in the indices are changed, the stock values in the indices are rebalanced as follows.
- The total value of each index maintains unchanged before and after rebalancing:
  $$V_{P}^{\text{total}}(t_{k}^{+}) = V_{P}^{\text{total}}(t_{k}^{-})$$ \hspace{1cm} (4)

  where:
  - $t_{k}^{-}, t_{k}^{+}$: the time before and after rebalancing
- The stock values after rebalancing are determined using the equal-weighted method.
The outcome of the SmartIndex method is a date-value series representing the performance of a firm’s positive, neutral, and negative rating indices. The following statistic measures are developed to evaluate the portfolios.

**Cumulative Return**

The cumulative return of an index at any date, \( t_k \), is equal to the index value at \( t_k \) divided by the initial index value, then minus 1.

\[
 r_p(t_1, t_k) = \left[ \frac{V_p^{total}(t_k)}{V_p^{total}(t_1)} - 1 \right] \cdot 100\% \tag{5}
\]

where:

- \( r_p(t_1, t_k) \): the cumulative return of the index

**Annualized Volatility**

The volatility is defined as the standard deviation of the index monthly return in the evaluation period multiplied by square root of 12.

\[
 \sigma_p^m = \sqrt{\frac{\sum_{M=1}^{N_m} \left[ r_p(M_i) - r_p^{avg} \right]^2}{N_m - 1}} \tag{6}
\]

\[
 \sigma_p^A = \sqrt{12} \cdot \sigma_p^m \tag{7}
\]

where:

- \( r_p(M_i) \): the index return in month \( i \)
- \( r_p^{avg} \): the average of the index monthly returns
- \( N_m \): # of months in the period
- \( \sigma_p^m \): the standard deviation of the index monthly return
- \( \sigma_p^A \): the annualized volatility of the index
Sharpe Ratio

The Sharpe Ratio is defined as the excess of the index annualized average monthly return over the annualized average monthly T-bill rate in the corresponding period divided by the annualized volatility of the index.

\[
SR_p(T) = \frac{r^A_p(T) - r^A_{tb}(T)}{\sigma^A_p(T)}
\]

where:

- \(SR_p\) : Sharpe Ratio of the index
- \(r^A_p\) : the annualized average monthly return of the index
- \(r^A_{tb}\) : the annualized average monthly return of the T-bill

Index Annual Turnover

It is equal to the total value bought or sold, whichever is less, during the evaluation period divided by the average index value. This number then is multiplied by 365 and divided by the number of calendar days in the timeframe.

\[
TO_p = \frac{365}{N_{\text{day}}} \cdot \min \left( \frac{\sum_{i=1}^{t_{\text{end}}} V^\text{buy}_p(t_i), \sum_{i=1}^{t_{\text{end}}} V^\text{sell}_p(t_i)}{N_{\text{day}} \sum_{i=1}^{t_{\text{end}}} V^\text{total}_p(t_i)} \right)
\]

where:

- \(TO_p\) : the annual turnover ratio of the index
- \(N_{\text{day}}\) : the number of calendar days in the evaluation period
- \(V^\text{buy}_p(t_i)\) : the stock value purchased at date \(t_i\) in the index
- \(V^\text{sell}_p(t_i)\) : the stock value sold at date \(t_i\) in the index